The Massachusetts Association of

Mathematics Leagues

and

The Actuaries´ Club of Boston

Announce

The Forty-Sixth Annual

Mathematics Olympiad Competition

First Level: **Thursday, October 22, 2009**

Second Level: **Tuesday, March 2, 2010**

Rules & Regulations

Purpose

- Stimulate interest in mathematics amongst secondary school students.
- Encourage achievement of high levels of proficiency in these schools.
- Acquaint the proficient student in mathematics with professional opportunities.

Scope

Elementary Number Theory, Algebra I, Algebra II, Plane Geometry, Solid Geometry, and Precalculus.

Eligibility

Official participation is open to students in grades 9-12 in Massachusetts.

Administration

The Executive Committee of the M.A.M.L. is the administrative agency for this competition.
Results of each examination will be forwarded to participating schools.

Prizes and Awards

- Only students in grades 9-12 are eligible for prizes, awards, and advancement to Level 2.
- The top scorer, including tie scores, in the First Level will receive the Martin J. Badoian Award. The top 10 scorers will also receive awards.
- The top scorer in each participating school will receive a Certificate, provided the score is above the 50th percentile and the student has received no other award.
- Finalist Certificates will be awarded to approximately 100 students who have the top scores on the First Level Examination. These finalists will be invited to participate in the Second Level Examination.
- Certificates of Merit will be awarded to approximately the next 150 top scorers.
- The following prizes are offered for the winners of the Second Level Examination:

First Prize	\$250
Second Prize	\$175
Third Prize	\$150
Fourth Prize	\$100
5th through 10th Prize	\$75
11th through 20th Prize	\$50

The top three students from Grades 9 or 10, excluding any in the top 20, receive a Book Award.

Sponsor

This competition is sponsored by the **Actuaries´ Club of Boston.**

First Level Examination

(i) Registration must be postmarked no later than **5 P.M. on Thursday, October 1, 2009, or a $25 late fee will be charged.**

(ii) There is a school Registration fee of $25. Individual examinations are purchased in bundles of 10 at a cost of $10 per bundle. Any late registration will be assessed an additional fee of $25. Check or money order should be made payable to: **MAML** and **<u>should accompany registration</u>**. If a purchase order is necessary, please include the **<u>purchase order number on the Registration Form.</u> Send *a copy* of this form to your business office. The Registration Form should be sent directly to the Contest Coordinator.**

(iii) Duration & Schedule: The First Level Examination will be a multiple choice examination of 90 minutes duration. Each school can decide its own starting time, beginning no earlier than 7:30 A.M. and no later than 9:00 A.M. on **Thursday, October 22, 2009**. **THERE IS NO FLEXIBILITY ALLOWED AS TO THIS TIME PERIOD,** and an administration at any other time would be considered "unofficial".

(iv) Proctoring and Scoring: Participating schools are required to obtain one of their own faculty members as examination supervisor. The supervisor will be responsible for the registration, the proctoring of the examination and the return of the answer sheets to the Contest Coordinator. The exams will be machine scored.

(v) Each school will receive the results of all its students participating in the First Level Examination.

Second Level Examination

(i) Duration & Schedule: The Second Level Examination will be three-hours in duration and will be conducted at examination centers in various sections of the state from 9:00 A.M. to 12:00 Noon on **Tuesday, March 2, 2010**.

(ii) Selection: Approximately 100 finalists will be chosen in strict order of rank based on results of the First Level Examination. Finalists will be notified of their selection by mail. Notification will include general information concerning the format of the Second Level Examination. Finalists will be assigned to an examination center.

The Officers of M.A.M.L. for the Year 2009–2010

President /Technology Coordinator
Michael Curry
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Acton 01720

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Worcester 01603

Four Sample Questions (Solutions are on an inserted page.)

1. The difference between the sum of 25 consecutive integers and the sum of the 10 consecutive integers that precede them is 1900. Find the largest of these numbers.

   (A) 93 (B) 95 (C) 126 (D) 127 (E) 129

2. How many three-digit numbers with exactly two repeating digits are a multiple of nine?

   (A) 36 (B) 21 (C) 20 (D) 18 (E) 12

3. Each side of a regular hexagon is 4 cm long. The midpoints of the sides are joined consecutively to form another hexagon. This process is continued forever. Determine the number of square centimeters in the total area of this infinite number of hexagons including the original one.

   (A) $8\sqrt{3}$ (B) $24\sqrt{3}$ (C) $42\sqrt{3}$ (D) $72\sqrt{3}$ (E) $96\sqrt{3}$

4. What is the sum of the rational values of x that satisfy the following equation:
   $$(x^2+4x+3)^{(x^2+4x+3)} = (x^2-3x-4)^{(x^2-3x-4)}$$

   (A) 1 (B) 0 (C) -1 (D) -3 (E) -4

1. Answer: **D.**

Thirty-five consecutive integers can be expressed as n, $n + 1$, $n + 2$, …, $n + 34$. The sum of the first 10 is $10n + 45$. The sum of the next 25 is $25n + 550$. The difference between these two quantities is $15n + 505$. Set this equal to 1900 and solve for n. We get $n = 93$. The largest of the numbers is $93 + 34 = 127$.

2. Answer: **B.**

A number that is divisible by 9 has the property that the sum of the digits is divisible by 9. Consider the repeating digits in order. 00, 11, 22, 33, 44, 55, 66, 77, 88, 99. The third digit that is included in the number is uniquely determined. In the case of 00, the third digit must be a 9. The only possibility is 900. In the case of 11, the third digit must be a 7. So, there are three possibilities, 117, 171, and, 711. Likewise there are three cases for three digit numbers that include 2-2-5, 4-4-1, 5-5-8, 7-7-4, and 8-8-2. In the case of 9-9-0, there are 2 possibilities, 990, and 909. We do not count the case of 3-3-3 and 6-6-6 since neither of these has exactly 2 repeating digits. So, the answer is $1+3+3+3+3+3+3+2 = 21$

3. Answer: **E**

When connecting two consecutive midpoints, an isosceles triangle is formed having a vertex angle of measure 120° and legs of measure 2 cm. Drawing the angle bisector of the vertex angle gives two 30-60-90° triangles with the result that the segment joining the midpoints is of length $2\sqrt{3}$. The area of the original hexagon, using the formula $\frac{3}{2}s^2\sqrt{3}$ is $24\sqrt{3}$ sq. cm. The area of the second hexagon is $18\sqrt{3}$ sq. cm. The ratio of the area of a hexagon to the next larger one is $\frac{3}{4}$. The areas of this collection of hexagons form an infinite geometric progression. The total area is $\frac{a}{1-r} = \frac{24\sqrt{3}}{1-\frac{3}{4}} = 96\sqrt{3}$ sq. cm.

4. Answer: **A.**

Factor all of the trinomials. $\left((x+1)(x+3)\right)^{(x-3)(x-1)} = \left((x-4)(x+1)\right)^{(x-1)(x+4)}$. If $x=-1$, we get $0^8 = 0^{-6}$, but the right side is 0^{-6}, which is undefined. If $x=1$, we get $8^0 = (-6)^0 \Rightarrow 1=1$, thus the sum of the rational solutions is 1.

In April, 1964, at the request of Roy Lane, then Head of the Mathematics Department, Hamilton High School, a group of representatives from each of many high schools in the North Shore area gathered to consider the possibility of a mathematics promotional program.

As a result of this meeting, study groups were formed. A second meeting was held at Concord-Carlisle High school to which representatives from schools in the various leagues throughout the State were invited to attend as were representatives from the Boston Actuaries Club and the Mathematics Department of the University of Massachusetts.

At this time, the University of Massachusetts Mathematics Department made it known hat it did not want to continue to have the responsibility of handling the competitive mathematics examination sponsored annually by the Actuaries Club. The idea of continuing he competition was foremost in the minds of the representatives at this meeting. An organization to be known as the Massachusetts Association of Mathematics Leagues was then considered and formed to deal with the problem. Lee Jameson, then of the Mathematics Department of Beverly High School, representing the Massachusetts Mathematics League, was appointed temporary chairman. John Waite, Jr. of Winchester High School, representing he Greater Boston Mathematics League, Professor Stephen Allen of the University of Massachusetts, William Favorite of Wilbraham Academy, representing the Western Massachusetts Interscholastic Mathematics League, Walter Luce of Tantasqua Regional High School, representing the Worcester County Mathematics League, and William Taylor of the Actuaries Club of Boston and associated with the Massachusetts Mutual Life Insurance Company of Springfield, composed the remainder of the Steering Committee. This committee met several times during the summer of 1964 at the Massachusetts Mutual Life Insurance Company in Springfield.

In September, 1964, the temporary committee resolved itself into the first Executive Board of the Massachusetts Association of Mathematics Leagues – M.A.M.L. An attorney from the Massachusetts Mutual Life Insurance Company was employed to advise the group of he legal aspects involved and to write the original by-laws. William Favorite was elected the chairman for a term of one year. John Waite, Jr. was elected the vice-chairman, and Lee Jameson was elected secretary-treasurer. By virtue of the fact that William Favorite was made chairman, it was deemed that he should become the first Contest Director. With the assistance of William Taylor, they set up dates and format for the first Annual Olympiad; the first in November was to be a statewide objective type test, while the second test to be given in April, 1965, to the 100 highest scoring students on the first test, was to be of the free-response type (five questions each having three parts, only one of these five questions to be based on topics chosen from outside the standard curriculum material).

Thus was the beginning of M.A.M.L. The idea of continuing competition on a statewide level in the area of mathematics was the underlying principle behind the formation of this association.