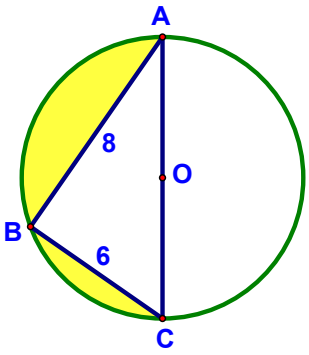
Ratios of Areas - Lesson 11-7

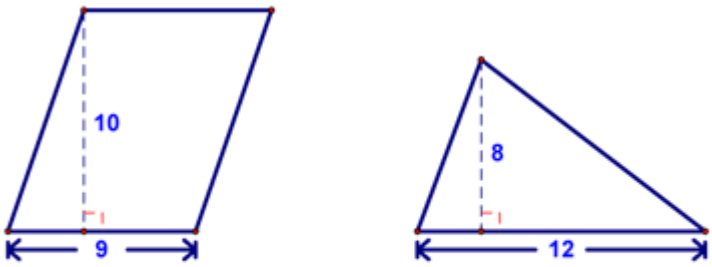
Warmup!

Given: ⊙O

Find: The area of the shaded region

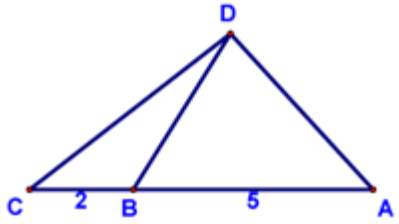


Today we're going to look at the ratio of areas of figures. We can start with the following example:



Find the ratio of the areas of the parallelogram to the triangle

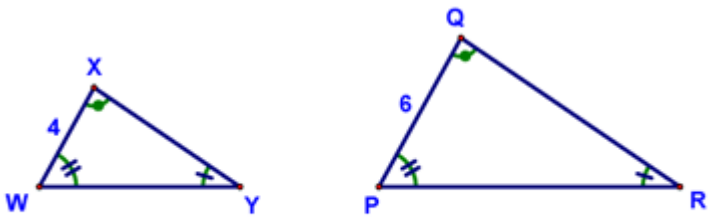
Now look at a second example:



Find the ratio of that area of △ABD to △CBD

Were you able to figure that one out? The key is that the height of each triangle is the same!

Next, consider the following example:



Given that △PQR ~ △WXY, find the ratio of their areas

This is interesting...all we have is similar triangles with the ratio of corresponding sides. However, that is enough information to solve the problem!

This fact turns out to hold true for all similar polygons:

Theorem 107

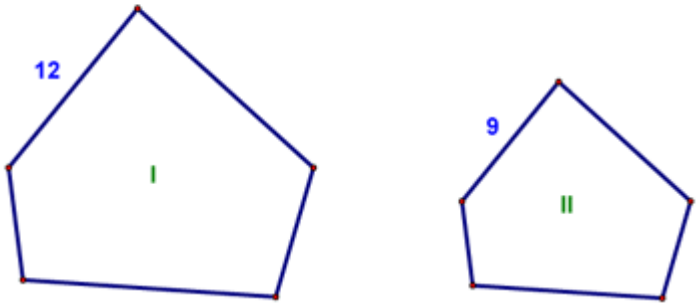
If two figures are similar, then the ratio of their areas equals the square of the ratio of their corresponding parts (*Similar-Figures Theorem*).

$$\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$$

where A_1 and A_2 are the areas and s_1 and s_2 are measures of the corresponding parts.

This theorem makes problems of this sort pretty easy. Here's an example:

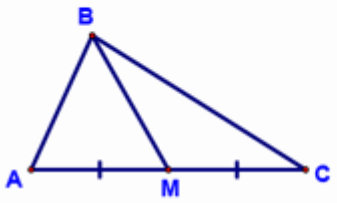
Given the similar polygons shown, find the ratio of their areas



There is one more theorem for today:

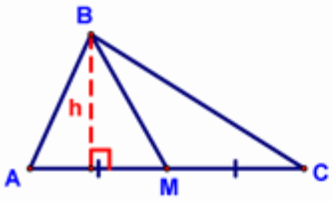
Theorem 108

A median of a triangle divides the triangle into two triangles with equal area (*Triangle Median Theorem*).



$A_{\triangle ABM} = A_{\triangle CBM}$

This should make sense to you if you think about it and look at the following:



$A_{\triangle ABM} = A_{\triangle CBM}$

$A_{\triangle ABM} = \frac{1}{2}(AM)h = A_{\triangle CBM} = \frac{1}{2}(CM)h$ since AM = CM