Dover-Sherborn High School
Mathematics Curriculum
Calculus Honors

A. DESCRIPTION

In the context of real scientific and mathematical questions, the collaborative experiences of this class will allow for the discovery of calculus through a series of student investigations and projects. Numerical methods and geometric visualizations will have central roles; differential equations that model dynamical systems will be scrutinized. The pace of the course is slower than that of the AP Calculus courses and allows for more time to fully understand the concepts. The outline below is intended to indicate the scope of the course, but it is not necessarily the order in which the topics will be taught.

B. OBJECTIVES

The student should be able to:

1. Analyze the fundamental planar curves (linear, quadratic, absolute value, cubic, exponential, logarithmic, trigonometric, and logistic) and their transformations represented both graphically and algebraically
2. Analyze limits of functions and continuity numerically, graphically and algebraically
3. Analyze the derivative as instantaneous rate of change, and as limit of the difference quotient, both at a point and as a function, to solve application problems
4. Compute derivatives of basic functions (power, exponential, logarithmic, inverse trigonometric), via Chain Rule and implicit differentiation
5. Use the integral as an accumulator of area to solve application problems
6. Evaluate definite integrals using the Fundamental Theorem of Calculus and by numerical approximation
7. Solve application problems involving derivatives, integrals and anti-derivatives
8. Use the functions of a graphing calculator to evaluate derivatives and integrals and apply these values to application problems

C. OUTLINE

1. Functions, Graphs, and Limits
    a. Analysis of graphs
    b. Limits of functions
        i. Understanding the limiting process
        ii. Calculating limits using algebra
        iii. Estimating limits from graphs or tables of data
    c. Asymptotic and unbounded behavior
        i. Graphical behavior of asymptotes
        ii. Asymptotic behavior in terms of limits involving infinity
        iii. Comparing relative magnitudes of functions and their rates of change (e.g., contrasting exponential growth, polynomial growth, and logarithmic growth)

d. Continuity as a property of functions
   i. Understanding continuity from an intuitive perspective
   ii. Understanding continuity in terms of limits
   iii. Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

2. Derivatives
   a. Concept of the derivative
      i. Graphical, numerical, and analytical representations of the derivative
      ii. Derivative interpreted as an instantaneous rate of change
      iii. Derivative defined as the limit of the difference quotient
      iv. Differentiability vs. Continuity
   b. Derivative at a point
      i. Slope of a curve at a point
      ii. Tangent line to a curve at a point and local linear approximation
      iii. Instantaneous rate of change as the limit of average rate of change
      iv. Approximate rate of change from graphs and tables of values.
   c. Derivative as a function
      i. Corresponding characteristics of graphs of f and f'
      ii. Relationship between the increasing and decreasing behavior of f and the sign of f'
      iii. The Mean Value Theorem and its geometric consequences
      iv. Equations involving derivatives.
   d. Second derivatives
      i. Corresponding characteristics of the graphs of f, f', and f''
      ii. Relationship between the concavity of f and the sign of f''
      iii. Points of inflection as places where concavity changes
   e. Applications of derivatives
      i. Analysis of curves, including the notions of monotonicity and concavity
      ii. Optimization (both absolute and relative extrema)
      iii. Modeling rates of change, including related rates problems
      iv. Use of implicit differentiation to find the derivative of an inverse function
      v. Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
      vi. Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
   f. Computation of derivatives
      i. Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
      ii. Basic rules for the derivative of sums, products, and quotients of functions
      iii. Chain rule and implicit differentiation

3. Integrals
   a. Interpretations and properties of definite integrals
      i. Definite integral as a limit of Riemann sums
      ii. Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_b^a f'(x)dx = f(b) - f(a)$$

      iii. Basic properties of definite integrals
   b. Applications of integrals
   c. Fundamental Theorem of Calculus
      i. Use of the Fundamental Theorem to evaluate definite integrals
      ii. Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined
   d. Techniques of antidifferentiation
      i. Antiderivatives following directly from derivatives of basic functions
      ii. Antiderivatives by substitution of variables
   e. Applications of antidifferentiation
      i. Finding specific antidervatives using initial conditions, including applications to motion along a line
      ii. Solving separable differential equations and using them in modeling (in particular, studying the equation $y' = ky$ and exponential growth)
   f. Numerical approximations to definite integrals

D. TEXT

Calculus, Finney, Demana, Waits, Kennedy (Addison Wesley, Longman, Inc., 1999)
ISBN: 0-201-32445-8

E. RESOURCE MATERIALS
1. Precalculus with Limits, Larson, Hostetler, Edwards (Houghton Mifflin, 1997) ISBN: 0-669-41758-0
2. Trigonometry, Hayden, Hall (Prentice Hall, 1990) ISBN: 0-13-930835-0
3. Graphing calculators and their manuals
4. Computer Programs
5. Web resources
6. Practice exams