***Dover-Sherborn High School***
***Mathematics Curriculum***
***AP Calculus BC***

A. DESCRIPTION

This course prepares students for the BC level AP Calculus exam. The syllabus adheres to the AP Calculus BC curriculum. Students taking this class are expected to sit for the AP exam. The outline below is intended to indicate the scope of the course, but it is not necessarily the order in which the topics will be taught.

B. OBJECTIVES

The student should be able to:

1. Analyze the fundamental planar curves (linear, quadratic, absolute value, cubic, exponential, logarithmic, trigonometric, and logistic) and their transformations represented both graphically and algebraically
2. Analyze limits of functions and continuity numerically, graphically and algebraically
3. Analyze the derivative as instantaneous rate of change, and as limit of the difference quotient, both at a point and as a function, to solve application problems
4. Compute derivatives of basic functions (power, exponential, logarithmic, inverse trigonometric ), via Chain Rule and implicit differentiation
5. Use the integral as an accumulator of area to solve application problems
6. Evaluate definite integrals using the Fundamental Theorem of Calculus and by numerical approximation
7. Solve application problems involving integrals and antiderivatives

C. OUTLINE

1. Functions, Graphs, and Limits
   a. Analysis of graphs
   b. Limits of functions
      i. Understanding the limiting process
      ii. Calculating limits using algebra
      iii. Estimating limits from graphs or tables of data
   c. Asymptotic and unbounded behavior
      i. Graphical behavior of asymptotes
      ii. Asymptotic behavior in terms of limits involving infinity
      iii. Comparing relative magnitudes of functions and their rates of change (e.g., contrasting exponential growth, polynomial growth, and logarithmic growth)
   d. Continuity as a property of functions
      i. Understanding continuity from an intuitive perspective
      ii. Understanding continuity in terms of limits
      iii. Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)
   e. Parametric, polar, and vector functions

2. Derivatives
    a. Concept of the derivative
        i. Graphical, numerical, and analytical representations of the derivative
        ii. Derivative interpreted as an instantaneous rate of change
        iii. Derivative defined as the limit of the difference quotient
        iv. Differentiability vs. Continuity
    b. Derivative at a point
        i. Slope of a curve at a point
        ii. Tangent line to a curve at a point and local linear approximation
        iii. Instantaneous rate of change as the limit of average rate of change
        iv. Approximate rate of change from graphs and tables of values.
    c. Derivative as a function
        i. Corresponding characteristics of graphs of $f$ and $f'$
        ii. Relationship between the increasing and decreasing behavior of $f$ and the sign of $f'$
        iii. The Mean Value Theorem and its geometric consequences
        iv. Equations involving derivatives.
    d. Second derivatives
        i. Corresponding characteristics of the graphs of $f$, $f'$, and $f''$
        ii. Relationship between the concavity of $f$ and the sign of $f''$
        iii. Points of inflection as places where concavity changes
    e. Applications of derivatives
        i. Analysis of curves, including the notions of monotonicity and concavity
        ii. Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration
        iii. Optimization (both absolute and relative extrema)
        iv. Modeling rates of change, including related rates problems
        v. Use of implicit differentiation to find the derivative of an inverse function
        vi. Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
        vii. Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
        viii. Numerical solution of differential equations using Euler's method
        ix. L'Hospital's rule, including its use in determining limits and convergence of improper integrals and series
    f. Computation of derivatives
        i. Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
        ii. Basic rules for the derivative of sums, products, and quotients of functions
        iii. Chain rule and implicit differentiation
        iv. Derivatives of parametric, polar, and vector functions

3. Integrals
    a. Interpretations and properties of definite integrals
        i. Definite integral as a limit of Riemann sums
        ii. Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:
        iii. $\int_b^a f'(x)dx = f(b) - f(a)$
        iv. Basic properties of definite integrals
    b. Applications of integrals
    c. Fundamental Theorem of Calculus
        i. Use of the Fundamental Theorem to evaluate definite integrals
        ii. Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined
    d. Techniques of antidifferentiation
        i. Antiderivatives following directly from derivatives of basic functions
        ii. Antiderivatives by substitution of variables, parts, and simple partial fractions
        iii. Improper integrals
    e. Applications of antidifferentiation
        i. Finding specific antidervatives using initial conditions, including applications to motion along a line
        ii. Solving separable differential equations and using them in modeling (in particular, studying the equation $y' = ky$ and exponential growth)
        iii. Solving logistic differential equations and using them in modeling
    f. Numerical approximations to definite integrals

4. Polynomial Approximations and Series
    a. Concept of series
    b. Series of constants
        i. Motivating examples, including decimal expansion
        ii. Geometric series with applications
        iii. The harmonic series
        iv. Alternating series with error bound
        v. Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of *p*-series
        vi. The ratio test for convergence and divergence
        vii. Comparing series to test for convergence or divergence

c. Taylor series
   i. Taylor polynomial approximations with graphical demonstration of convergence
   ii. Maclaurin series and the general Taylor series centered at $x = a$
   iii. Maclaurin series for the functions $e^x$, $\sin x$, $\cos x$, and $\frac{1}{1-x}$
   iv. Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, and antidifferentiation, and the formation of new series from known series
   v. Functions defined by power series
   vi. Radius and interval of convergence of power series
   vii. Lagrange error bound for Taylor polynomials

D. TEXT

Calculus, Finney, Demana, Waits, Kennedy (Addison Wesley, Longman, Inc., 1999)
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E. RESOURCE MATERIALS
1. Graphing calculators
2. Computer Programs
3. Web resources
4. Practice exams